

Oscillating universe with quintom matter

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Abstract

In this paper, we study the possibility of building a model of the oscillating universe with quintom matter in the framework of 4-dimensional Friedmann-Robertson-Walker background. Taking the two-scalar-field quintom model as an example, we find in the model parameter space there are five different types of solutions which correspond to: (I) a cyclic universe with the minimal and maximal values of the scale factor remaining the same in every cycle, (II) an oscillating universe with its minimal and maximal values of the scale factor increasing cycle by cycle, (III) an oscillating universe with its scale factor always increasing, (IV) an oscillating universe with its minimal and maximal values of the scale factor decreasing cycle by cycle, and (V) an oscillating universe with its scale factor always decreasing.

1 Introduction

The quintom scenario of dark energy firstly proposed in Ref. [1] for the purpose of understanding the dynamical feature with the equation-of-state (EoS) crossing over the cosmological constant boundary $w = -1$ differs from the quintessence or phantom or other scenario of dark energy in the determination of the evolution of the Universe. In Ref. [2], four of us (Cai, Qiu, Piao and Zhang) have considered an application of the quintom matter in the early universe and interestingly we have found a bouncing solution within the framework of the standard 4-dimensional Friedmann-Robertson-Walker (FRW) background. In our model the background evolution can be studied analytically and numerically. Later on its perturbation theory has been developed in Refs. [3, 4] and it is found that the perturbations of this model possess some features of both singular bounce and non-singular bounce models. In this paper we extend our study to constructing a model of oscillating universe with the quintom matter.

The idea of cyclic universe was initially introduced in 1930's by Richard Tolman [5]. Since then there have been various proposals in the literature. The authors of Refs. [6, 7] introduced a cyclic model in high dimensional string theory with an infinite and flat universe. With a modified Friedmann equation the cyclic evolution of the universe can also be realized [8, 9]. In Ref. [10] four of us (Xiong, Qiu, Cai and Zhang) realized that in the framework of loop quantum cosmology (LQC) a cyclic universe can be obtained with the quintom matter. In this paper, however, we will study the solution of oscillating universe in the absence of the modification of the standard 4-dimensional Einstein Gravity with a flat universe.

To begin with, let us examine in detail the conditions required for an oscillating solution. The basic picture for the evolution of the cyclic universe can be shown below:

$$\dots \text{bounce} \xrightarrow{\text{expanding}} \text{turn-around} \xrightarrow{\text{contracting}} \text{bounce} \dots \quad (1)$$

In the 4-dimensional FRW framework the Einstein equations can be written down as:

$$H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{\rho}{3M_p^2} \quad \text{and} \quad \frac{\ddot{a}}{a} = -\frac{\rho + 3p}{6M_p^2}, \quad (2)$$

where we define $M_p^2 \equiv \frac{1}{8\pi G}$, H stands for the Hubble parameter, while ρ and p represent the energy density and the pressure of the universe respectively. By definition,

for a pivot (bounce or turn-around) process to occur, one must require that at the pivot point $\dot{a} = 0$ and $\ddot{a} > 0$ around the bouncing point, while $\ddot{a} < 0$ around the turn-around point. According to Eq. (2), one can get

$$\rho = 0, \quad p < 0 \quad (\text{or } p > 0) \quad \text{for the bounce (or turn-around),} \quad (3)$$

or equivalently, $w \equiv \frac{p}{\rho} \rightarrow -\infty$ (or $+\infty$) at the bounce (or turn-around) point with the parameter w being the EoS of the matter filled in the universe. We can see that, when the universe undergoes from bounce to turn-around, the EoS of the matter evolves from $-\infty$ to $+\infty$; while in the converse case, the EoS goes from $+\infty$ to $-\infty$. That shows w needs to cross over the cosmological constant boundary ($w = -1$) in these processes, which interestingly implies the necessity to have the quintom matter for the realization of the oscillating universe under the 4-dimensional Einstein Gravity.

Another interesting evolution of an oscillating universe is that, this universe undergoes accelerations periodically. In this scenario, we are able to unify the early inflation and current acceleration of the universe, leading to the oscillations of the Hubble constant and a recurring universe. During this kind of evolution, the universe would not encounter a big crunch nor big rip. The scale factor keeps increasing from one period to another and so leads to a highly flat universe naturally. This scenario was firstly proposed by Ref. [11] in which a parameterized Quintom model was used, and in that paper the coincidence problem was argued to be reconciled.

In this paper we will take the two-scalar-field quintom model for a detailed study on the oscillating universe. We will show that for those models considered in Ref. [2] with a positive-definite potential there is no oscillating solution, but it happens when allowing the potential to possess negative regions. As is proven below, for the two-scalar-field quintom models, a negative potential is necessary for the scenario of a cyclic universe. Our paper is organized as follows. In section 2, we provide an exact solution of the oscillating universe in the quintom model of two-scalar-field and study the trajectory of the fields in the phase space. Moreover, we study various possible evolutions of the universe in our model. Section 3 contains the conclusion and discussions.

2 A solution of oscillating universe in the two-scalar-field quintom model

2.1 The model

The simplest quintom model consists of two scalars with one being quintessence-like and another the phantom-like. Its action is given by

$$S = \int d^4x \sqrt{-g} \left[\frac{1}{2} \partial_\mu \phi \partial^\mu \phi - \frac{1}{2} \partial_\mu \psi \partial^\mu \psi - V(\phi, \psi) \right], \quad (4)$$

where the metric is in form of $(+, -, -, -)$. In the framework of FRW cosmology, we can easily obtain the energy density and the pressure of the model,

$$\rho = \frac{1}{2} \dot{\phi}^2 - \frac{1}{2} \dot{\psi}^2 + V(\phi, \psi) \quad , \quad p = \frac{1}{2} \dot{\phi}^2 - \frac{1}{2} \dot{\psi}^2 - V(\phi, \psi) \quad , \quad (5)$$

and the equations of motion for these two fields,

$$\ddot{\phi} + 3H\dot{\phi} + V_{,\phi} = 0 \quad , \quad \ddot{\psi} + 3H\dot{\psi} - V_{,\psi} = 0 \quad . \quad (6)$$

Phenomenologically, a general form of the potential for a renormalizable model includes operators with dimension 4 or less including various power form of the scalar fields. For the study of this paper we impose a Z_2 symmetry on this model, *i.e.*, the potential will keep invariant under the transformations $\phi \rightarrow -\phi$ and $\psi \rightarrow -\psi$ simultaneously. Then the potential of the model is given by

$$\begin{aligned} V(\phi, \psi) = & V_0 + \frac{1}{2} m_1^2 \phi^2 + \frac{1}{2} m_2^2 \psi^2 + \gamma_1 \phi^4 + \gamma_2 \psi^4 \\ & + g_1 \phi \psi + g_2 \phi \psi^3 + g_3 \phi^2 \psi^2 + g_4 \phi^3 \psi. \end{aligned} \quad (7)$$

From the condition (3), we can see that at both the bounce and the turn-around point $\dot{\psi}^2 = \dot{\phi}^2 + 2V$. From Eq. (5) we have $p = -2V$. When the universe undergoes a bounce the pressure is required to be negative, which implies the potential to be positive. However, when a turn-around solution takes place, the pressure of the universe is required to be positive, consequently the potential must be negative. The argument above shows the expected quintom model needs to contain a negative term in its potential to realize the oscillating scenario.

In our scenario the two scalar fields dominate the universe alternately. To show it, let us assume the universe starts an expansion from a bounce point without losing

generality. We find that, initially the universe is phantom dominated and the energy density of the universe grows up. However, since we need to enter a quintessence dominated stage, the energy density has to reach a maximal value and then decreases. When it arrives at zero, the turn-around happens and the universe enters into a contracting phase. Therefore, from the bounce to the turn-around the universe needs to transit from the phantom dominating phase into the quintessence dominating phase, or vice versa. In order to describe the whole evolution explicitly, we in Fig. 1 sketch the evolutions for the energy density and the scale factor of the cyclic universe in our model. We can see that during each cycle the universe is dominated by quintessence ϕ and phantom ψ alternately. However, as is pointed out in Refs. [12, 13], this process will not happen if the two fields are decoupled. Therefore, in our model an interaction between the two fields is also crucial.

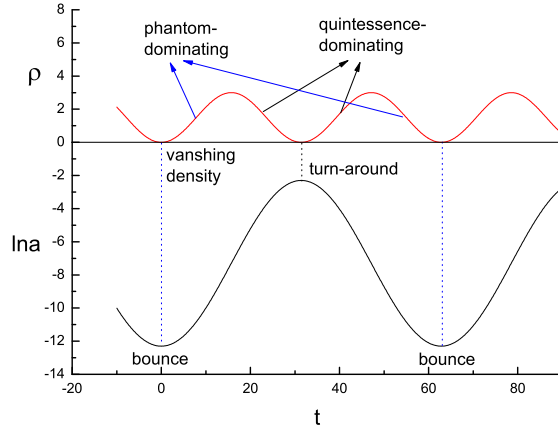


Figure 1: This sketch plot shows the evolution of the energy density in a cyclic universe. For each cycle the quintessence-like and phantom-like components dominate alternately.

For a detailed quantitative study we take the following form of the potential

$$V(\phi, \psi) = (\Lambda_0 + \lambda\phi\psi)^2 + \frac{1}{2}m^2\phi^2 - \frac{1}{2}m^2\psi^2, \quad (8)$$

where λ is a dimensionless constant describing the interaction between two scalar fields, and Λ_0 an constant with dimension of $[\text{mass}]^2$.

This potential takes a negative value when ϕ is near the origin and ψ with a large value. However, due to the interaction between these two fields, the potential is still

bounded from below and is positive definite when the fields are both away from the center, especially for $\lambda\phi > \frac{\sqrt{2}}{2}m$.

With the potential (8) we solve the equations of motion (6). Inserting the energy density and pressure (5) into the Friedmann equations (2), we find a solution where ϕ and ψ are given by:

$$\phi = \sqrt{A_0} \cos mt, \quad \psi = \sqrt{A_0} \sin mt, \quad (9)$$

where the parameter A_0 describes the oscillating amplitude of the fields (also with dimension of $[\text{mass}]^2$). Besides, we obtain that for this solution $\lambda = \frac{\sqrt{3}m}{2M_p}$, thus the detailed behavior of the solution is characterized by the three free parameters which are m, Λ_0, A_0 respectively.

From Eq. (9), we can learn that the two scalar fields oscillate, however with a phase difference $\pi/2$. So the fields ϕ and ψ dominate the universe alternately, which is what we have analyzed previously.

2.2 Classifications of the solutions

In this section, we study the detailed cosmological evolutions of this model. We will see that this model gives rise to five different scenarios of oscillating cosmology within the model parameter space.

First of all, from the Einstein Equation (2) one can easily get the Hubble parameter as follows,

$$H = \frac{\sqrt{3}}{3M_p} (\Lambda_0 + \Lambda_1 \sin 2mt), \quad (10)$$

where we define $\Lambda_1 \equiv \frac{\sqrt{3}m}{4M_p} A_0$. For different parameters, the evolution of the universe can be classified into five cases. The first one describes an exactly cyclic universe with its scale factor oscillating periodically. The second one is that the scale factor of the universe oscillates with the center value increasing gradually. In the third one, the Hubble parameter is oscillating periodically but always positive, correspondingly the scale factor is always increasing during the whole period of evolution. The fourth one can be viewed as a reverse process of the second one since in that case the center value of the scale factor is decreasing gradually. The last case describes the reverse process of the third one, and in this case, the scale factor always decreases and so it

corresponds to a nonphysical one. In the following we will study in detail these five cases one by one. Note that we will take the natural unit $M_p = 1$ in the following calculations.

Case (I): $\Lambda_0 = 0$. In this case the Hubble parameter is given by $H = \frac{mA_0}{4M_p^2} \sin 2mt$. So for the scale factor we have

$$\ln a \propto \cos 2mt . \quad (11)$$

This gives rise to a cyclic universe with the minimal and maximal values of the scale factor remaining the same in every cycle. In this scenario there is no spacetime singularity. In Fig. 2 we plot the evolutions of the scale factor, the Hubble parameter, the energy density and the EoS.

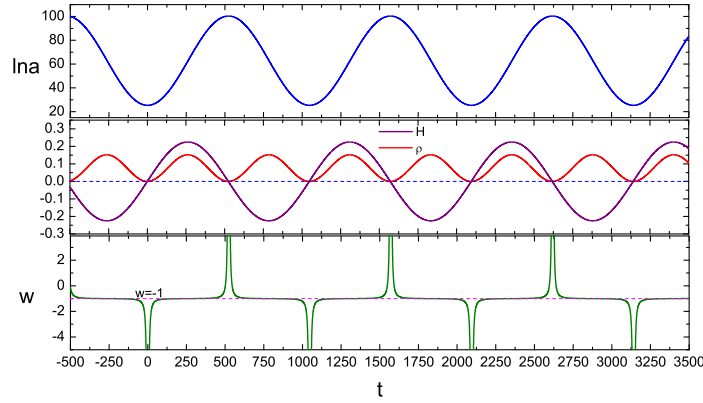


Figure 2: The plot of the evolutions for Case (I). This plot shows an exactly cyclic universe. The scale factor of the universe oscillates between the minimal and maximal value. In the numerical calculation we take $m = 3 \times 10^{-3}$ and $A_0 = 300$ or equivalently $\Lambda_1 \simeq 0.39$. The Hubble parameter is shown by the purple line, and the energy density is described by the red line.

To show this solution more explicitly, we give the potential V as a function of fields ϕ and ψ in Fig. 3. It is easy to see that the potential is bounded from below in the phase space when ϕ is far away from the origin, and only the center region of the surface is a hyperbolic paraboloid. In the right panel of Fig. 3, we also give the trajectories of the fields ϕ and ψ on the surface of the potential. From Fig. 3 we

read that the projection of the trajectory on the field plane (ϕ, ψ) is a circle which is consistent with the solution (9).

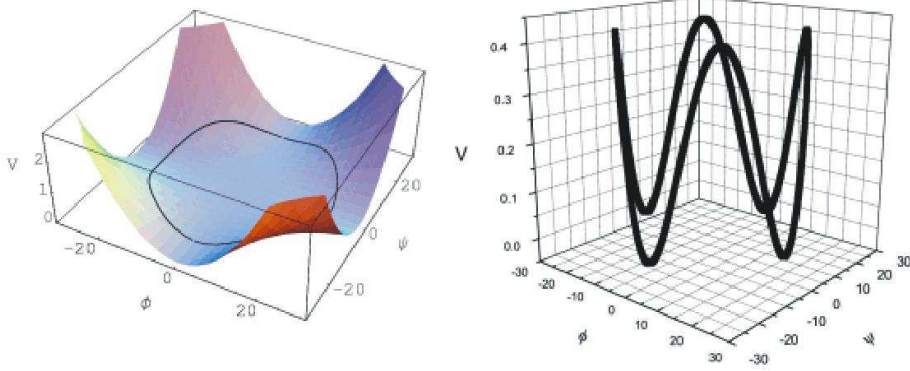


Figure 3: The left panel is the potential $V(\phi, \psi)$ with respect to the fields ϕ and ψ , and the right panel is a closer view of the trajectory of the fields on the potential surface. The saddle point lies on the zero point in the phase space. The black solid line shows the trajectory of the fields ϕ and ψ on the surface of the potential.

Case (II): $0 < \Lambda_0 < \Lambda_1$. In this case the scale factor is solved as

$$\ln a \propto C_1 t + C_2 \cos 2mt, \quad (12)$$

where C_1 and C_2 are constants, $C_1 = \Lambda_0/\sqrt{3}M_p$, $C_2 = -A_0/8M_p^2$. The evolution picture is shown in Fig. 4. The solution also describes a cyclic universe. However, different from Case (I), as the universe evolves both the minimal and maximal values of scale factor increase cycle by cycle. Therefore, the average size of the universe is still growing up gradually without big crunch or big rip singularities, although its scale factor experiences contractions and expansions alternately. With the backward evolution the scale factor can not shrink to zero in finite time.

Case (III): $\Lambda_0 \geq \Lambda_1$. The solution for the scale factor is the same as in Case (II). However the evolution of the universe is different. In this case the universe lies in the expanding period forever and there is no contracting phase. An interesting point is that the accelerating expansion of the universe is periodical. This is shown by the oscillating EoS in Fig. 5. Since the EoS w is oscillating around “-1”, in the evolution the big rip can be avoided. For reasonable parameters, we will be able to unify the inflationary period and the late time acceleration together in this case.

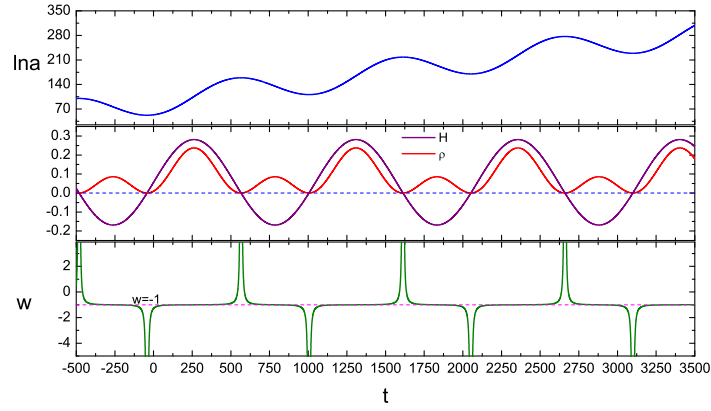


Figure 4: The plot of the evolutions for Case (II). We take the values of m and A_0 to be the same as those used in Fig. 2, and $\Lambda_0 = 0.10$ which satisfies the condition $0 < \Lambda_0 < \Lambda_1$. This figure also describes an oscillating evolution of the universe, of which the average value of scale factor is increasing during the evolution. The figure for energy density and Hubble parameter is like Fig. 2, however, the difference is that the oscillating amplitude of the energy density in the expanding period is larger than in the contracting phase.

This possibility of oscillating quintom had been considered in Ref. [11], however the concrete field model is not provided there.

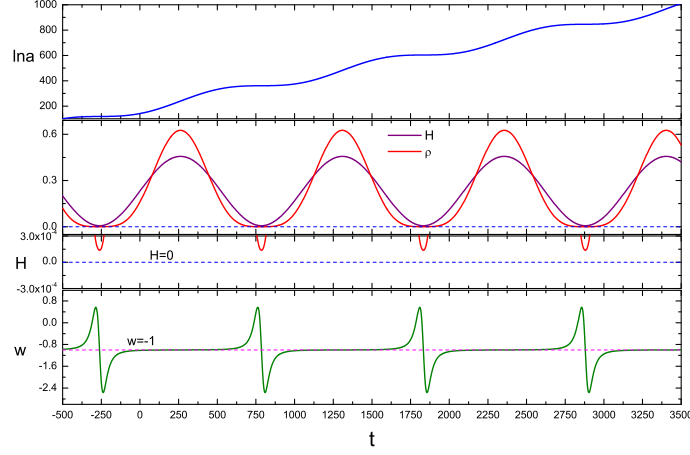


Figure 5: The plot of the evolutions for Case (III). The coefficients m and A_0 are taken the same values as in Fig. 2, and $\Lambda_0 = 0.402$ which satisfies the condition $\Lambda_0 \geq \Lambda_1$. This figure shows that the universe is expanding all the time, without any bounce or turn-around. For such a universe the energy density and Hubble parameter are always positive. An interesting feature of this scenario is that the accelerating expansion of the universe is periodical. This plot also shows the oscillating EoS around “ -1 ”.

Case (IV): $-|\Lambda_1| < \Lambda_0 < 0$. This case corresponds to a cyclic universe with decreasing minimal and maximal scale factor for each epoch. Contrary to Case (II), the total tendency is that the scale factor decreases with the forward cycle. Fig. 6 shows the evolution picture. Moreover, with the forward evolution the scale factor can not decrease to zero value and reach the singularity in finite time.

Case (V): $\Lambda_0 < -|\Lambda_1|$. This case describes a contracting universe for all the time, which corresponds to the reverse of Case (III). Unfortunately, such a scenario do not give an expanding evolution and so has already been ruled out by observations.

Up to now, we have learned that there are mainly five types of evolutions for our model. Here we would like to naively comment on the relative merits of these solutions and their physical significance. First of all, solution (I) shows an exact cyclic

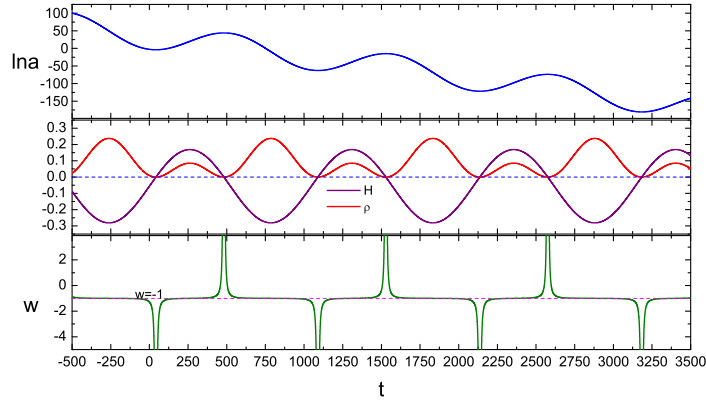


Figure 6: The plot of the evolutions for Case (IV). The values of m and A_0 are also the same as in Fig. 2, and $\Lambda_0 = -0.10$ which satisfies the requirement $-|\Lambda_1| < \Lambda_0 < 0$. This figure shows the oscillating evolution which is contrary to the Fig. 4. In such an oscillating universe the period of contracting phase is longer than the expanding phase, so the average value of the scale factor is decreasing as the universe evolves.

universe and the minimal value of its scale factor does not vanish, so this solution give the promise of the universe being singularity-free during the whole evolution. Moreover, for solutions (II) and (IV), although the minimal value of the scale factor is approaching to zero in the early time or late time respectively, this process needs infinite long time. Besides, since in these two solutions the null energy condition can be violated by Quintom, there can be no ending point for the geodesic curve. Therefore, we might say that both the big bang and big crunch singularities could be avoided in the these three cases. They can be viewed as one type of oscillating universe in which the scale factor performs an oscillating behavior. Furthermore, solution (III) gives another interesting oscillating scenario of the universe, that is, its Hubble parameter oscillates and keeps positive. In this scenario, the universe undergoes inflationary epoch periodically. So it explores an interesting possibility to unify the early inflation and late time acceleration of the universe. We do not discuss the last case for it is nonphysical.

2.3 Constraints on the model parameters

From the above analysis we know that the possible evolution of the universe depends upon the value of the constant Λ_0 in the potential. For different values of Λ_0 there exist four physical possibilities. Three of them contain expansions and contractions, so describe an cyclic universe. The other one depicts an forever expanding universe with an EoS oscillating around the cosmological constant boundary. All of these scenarios of the cosmological evolutions are free of singularity.

One may notice that the EoS of our model oscillates around the cosmological constant boundary, and every time when it approaches $w = -1$ it stays there for a while. Correspondingly, the Hubble parameter evolves near its maximal value and makes the scale factor expand exponentially. We treat this period as an inflationary stage after the bounce. Therefore, in our model there exists an inflationary period in each cycle. This is an important process, since the relic matters created in the last cycle could be washed out during this stage. In this process entropy can be diluted by inflationary epoch as well, so there is no necessary to worry about the infinite increase of entropy. Moreover, some of the primordial perturbations are able to exit the horizon, and reenter the horizon when the inflationary stage ceases. When these perturbations reenters, new structures in the next period will be formed.

Now we study the possible constraints on the model parameters (m, Λ_0, A_0) . First of all, we require the period of the oscillation of the Hubble parameter be no less than 2 times the age of our universe which is of order of the present Hubble time. So from Eq. (10) we can get the period $T = \frac{\pi}{m} \sim \mathcal{O}(H_0^{-1})$ where H_0^{-1} is the Hubble time and $H_0^{-1} \simeq 10^{60} M_p^{-1}$, so $m \sim \mathcal{O}(10^{-60}) M_p$. Secondly we require the maximal value of the Hubble parameter be able to reach the inflationary energy scale. The maximal value of H is given by $(\Lambda_0 + \Lambda_1)$ when $(\sin 2mt)$ reaches its maximum. For case (I), (III) and (IV), $|\Lambda_0| \leq \Lambda_1$, so we have $(\Lambda_0 + \Lambda_1) \sim \mathcal{O}(\frac{mA_0}{M_p^2})$. If we consider inflation with energy density, for example around $\mathcal{O}(10^{-20}) M_p^4$, we find A_0 must be of $\mathcal{O}(10^{50}) M_p^2$. Such a large value of A_0 indicates the scalar fields ϕ and ψ take values much beyond the Planck scale, which makes our effective lagrangian description invalid. One possibility of solving this problem is to consider a model with a large number of quintom fields. In this case the Hubble parameter is amplified by a pre-factor \sqrt{N} with N the number of quintom fields. If N is larger than or the same order of 10^{50} , A_0 can be relaxed to

be $\mathcal{O}(1)M_p^2$ or less.

3 Conclusion and discussions

In this paper, we have studied the possibility of constructing an oscillating solution of the universe in the two-scalar-field quintom model. Our results show that this model gives rise to cyclic cosmology where a universe expands and contracts alternately without singularity. Within the model parameter space we also provide a new kind of evolution for the oscillating universe, i.e, expanding forever with its EoS oscillating around -1 which leads to the universe accelerating periodically.

Cyclic universe scenario has been widely studied in the literature. Among them, there are cyclic models in the braneworld scenario [7, 9], closed oscillating universe [14], cyclic universe in Loop Quantum Cosmology [15, 16, 10], and see Refs. [17, 18, 19, 20, 21, 22, 23, 24, 25] for recent developments on kinds of oscillating universes. The main difference of our work presented in this paper from theirs is that the model we considered is restricted within the framework of 4-dimensional Einstein Gravity in a flat universe.

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